

Problem set 3

Mathematical Methods

Problem 1

Find the local extrema of the following problems and Check the second order condition to determine the type of extrema.

1) $z = x^2 + 2xy + 2y^2 - 2x - y$

2) $z = (y - x^2)(y - 2x^2)$

3) $f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ on $\{(x, y, z): x^2 + y^2 + z^2 = 1\}$.

4) $f(x, y) = -y$ on $\{(x, y): y^3 - x^2 = 0\}$

Problem 2

A consumer has income $I > 0$, she faces a price vector $P \in \mathbb{R}_+^3$ for the three commodities she consumes. All commodities must be consumed in nonnegative amounts. Moreover, she must consume at least two units of commodity 2 and cannot consume more than one unit of commodity 1. Assuming $I = 4$ and $P = (1, 1, 1)$, calculate the optimal consumption bundle if the utility function is given by $u(x_1, x_2, x_3) = x_1 x_2 x_3$.

Problem 3

Characterize the second order condition(s) for the standard consumer problem:

$$\begin{aligned} \max_{0 \leq x \leq 1} \quad & u(x) \\ \text{s. t. } & p \cdot x \leq I \end{aligned}$$

For the following two cases:

i) $x = (x_1, x_2)$.

ii) $x = (x_1, x_2, x_3)$.