Institute for Management and Planning Studies (IMPS)

Advanced Micro II, part 2, General Equilibrium Theory

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Problem set 5

More than one Equilibrium

Exercise 5.1

Consider an Edgeworth-box economy with two agents, whose preferences are characterized by the quasilinear utility functions

$$u_1(x_{11}, x_{21}) = x_{11} - \frac{1}{6}x_{21}^{-6}$$

$$u_2(x_{12}, x_{22}) = -\frac{1}{6}x_{12}^{-6} + x_{22}$$

Endowments are $\omega_1 = (2, r)$, $\omega_2 = (r, 2)$ with $r = 2^{\frac{6}{7}} - 2^{\frac{1}{7}}$.

(i) Find the offer curves[1] of both agents.

(ii) Show that the price ratios $\frac{p_1}{p_2} = 2, 1$ and $\frac{1}{2}$ constitute equilibrium price ratios.

(iii) Draw a figure.

Discontinuous Demand

Exercise 5.2

Let $[0, 1]^2 = [0, 1] \times [0, 1]$ be the unit square. The binary relation $\succ_L$ on $[0, 1]^2$ is defined by

$$(x_1, x_2) \succ_L (y_1, y_2) :\Leftrightarrow x_1 > y_1 \text{ or } [x_1 = y_1 \text{ and } x_2 > y_2].$$

(i) Show that $\succsim_L$ defined by $x \succsim_L y :\Leftrightarrow y \not\succ_L x$ defines a preference relation (reflexive, transitive, complete).

(ii) Show that there exists no utility function representing this preference relation.

[1] Remember, the offer curve of an agent is the utility maximizing allocation for a given endowment and for variable price ratios under the supposition of price taking behavior.

(iii) Let $\omega = (\frac{1}{2}, \frac{1}{2})$ be some household's endowment and

$$\widetilde{B}(p) = \{x \in [0,1]^2 | p \cdot x \leq p \cdot \omega\}$$

the bounded budget set for given prices $p \in P = \{p \in \mathbb{R}^2_+ | p_1 + p_2 = 1\}$. Show that demand

$$\widetilde{D}(p) = \{x \in \widetilde{B}(p) | x \succsim_L y \quad \forall y \in \widetilde{B}(p)\}$$

is discontinuous in p. Hint: It helps to consider convergent sequences of prices $p^\nu \to (1, 0)$.

State-Contingent Commodity

Exercise 5.3
Consider an economy with two agents, one physical good, and two states of the world. The utility function of agent i is given by $\pi_{1i}u(x_{1i}) + \pi_{2i}u(x_{2i})$, where π_{si} is agent i's subjective probability of state $s \in \{1, 2\}$. The marginal rate of substitution is given by

$$\frac{\partial x_{2i}}{\partial x_{1i}} = -\frac{\pi_{1i}u'(x_{1i})}{\pi_{2i}u'(x_{2i})}$$

or $-\frac{\pi_{1i}}{\pi_{2i}}$ for $x_{1i} = x_{2i}$.

(i) Suppose agents perceive the same probability distributions over states, i.e. $\pi_{11} = \pi_{12}$. Determine graphically the market equilibrium. Accordingly, determine the location of the market equilibrium without aggregate risk[2] in an Edgeworth box.

(ii) How do these observations change with no aggregate risk but different probability distributions?

(iii) Now consider the case with aggregate risk and the same probability distributions. Suppose that the economy's endowment is larger in state 1 than in state 2, i.e. $\omega_1 > \omega_2$. What can be said in this case about the price of the good in different states?

(iv) Finally suppose one of the agents is risk neutral. How can you describe a risk neutral decision maker? Characterize graphically the Pareto efficient risk sharing.

(v) Interpret your results.

[2] Equilibrium with no aggregate risk means that the economy's endowment of x coincides in both states of nature ($\omega_1 = \omega_2$).