

Problem set 2

Mathematical Methods

Problem 1

Let $M = I - X(X'X)^{-1}X'$.

- i) Show that M is idempotent: $MM = M$.
- ii) Show that M is symmetric: $M' = M$.
- iii) Denote: If X and X' are not square, we cannot assume that $(X'X)^{-1} = (X')^{-1}(X)^{-1}$. Must M be square? What about $X'X$ and X ?

Problem 2

Show the following relations (if they are true):

- i) $tr(I_n) = n$
- ii) $tr(ABA^{-1}) = tr(B)$
- iii) $tr(ABCD) = tr(DABC)$
- iv) $tr(ABC) = tr(ACB)$

Problem 3

Assume $n \geq 2$. Find the determinant for:

$$\begin{pmatrix} x & 1 & & 1 \\ 1 & x & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & x \end{pmatrix}$$

Problem 4

Calculate the eigenvalues and eigenvectors for the matrix:

$$\begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

Problem 5

Calculate a matrix V for the matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 3 & -1 & 3 \end{pmatrix}$$

, such that $V^{-1}AV = \Lambda$, a diagonal matrix.

Problem 6

Which of the following matrices are positive (semi)definite, negative (semi)definite, indefinite? For indefiniteness give an example-vector.

$$\begin{pmatrix} -4 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Problem 7

- i) Find a hyperplane (here a line) to separate (2,2) from the unit circle:
 $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$
- ii) Is there any hyperplane to separate the set $S = \{(x, y) \in \mathbb{R}^2 : x \geq (y - 1)^2\}$ and the point (-1,0).
- iii) For the above examples, provide P and α for which $P \cdot x \geq \alpha, \forall x \in S$ and the point does not satisfy this condition.