Institute for Management and Planning Studies (IMPS)

Advanced Micro II, part 2, General Equilibrium Theory

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Problem set 6

Price Adjustment Function

Exercise 6.1

In the lecture we used the mapping $T : P \longrightarrow P$ as a price adjustment function whose fixed points are competitive equilibria. Consider instead using the mapping $Q : P \longrightarrow P$, where the ith coordinate mapping of Q is given by

$$Q_i(p) = \frac{\max\{0, p_i + p_i \tilde{Z}_i(p)\}}{\sum_{j=1}^{L} \max\{0, p_j + p_j \tilde{Z}_j(p)\}}.$$

Assume that Walras' Law holds on equality, that is $p \cdot \tilde{Z}(p) = 0$.

(i) Show that every competitive equilibrium price vector p^0 is a fixed point of Q.

(ii) Show that every vertex of the price simplex P is also a fixed point of Q.

(iii) Under suitably chosen sufficient conditions on the economy, $Q(\cdot)$ can be shown to have a fixed point $p^* = Q(p^*)$. Does this prove that the economy – under those sufficient conditions – has a competitive equilibrium?

Core

Exercise 6.2

Consider a pure exchange economy $\mathcal{O}$ with 4 households and two goods. All households have identical preferences specified by the utility function

$$u(x_1, x_2) = x_1 x_2.$$

Initial endowments are given by $\omega^1 = \omega^2 = (10, 10)$ and $\omega^3 = \omega^4 = (10, 30)$. Check properties (1) feasability (2) Pareto-efficiency (3) element of the core (4) Walras-equilibrium for the following allocations $x = (x^1, ..., x^4)$:

(i) $x^1 = x^2 = \left(\frac{15}{2}, 15\right), x^3 = x^4 = \left(\frac{25}{2}, 25\right)$

(ii) $x^1 = x^2 = \left(\sqrt{50}, 2\sqrt{50}\right), x^3 = x^4 = \left(20 - \sqrt{50}, 40 - 2\sqrt{50}\right)$

(iii) $x^1 = \left(\sqrt{50}, 2\sqrt{50}\right), x^2 = \left(\frac{15}{2}, 15\right), x^3 = x^4 = \left(\frac{25}{2}, 25\right)$

(iv) $x^1 = (8, 12), x^2 = (9, 11), x^3 = (12, 23), x^4 = (11, 29)$

Core Equivalence

Exercise 6.3
Consider Edgeworthbox-exchange economy $\mathcal{O}$ with two types of households $u_1(x_1, x_2) = x_1 x_2$ with initial endowment $\omega^1 = (1, 9)$ and $u_2(x_1, x_2) = x_1 x_2$ with endowment $\omega^2 = (9, 1)$.

(i) Find an element of $C_1 \setminus C_2$, where C_N is the core of the $N-$replicator economy $\mathcal{O}^N$ as defined in the lecture.

(ii) Find an element of $C_2 \setminus C_3$ (calculation or graphical illustration).

(iii) Figure out the set of allocations in $\bigcap_{N=1,2,\ldots} C_N$.