Microeconomics II

Part I: Contract Theory

Hidden information

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2- Hidden Information

Hidden information: Some important information is hidden from principal.

This information asymmetry may be something like Agent's endowment is observable only by Agent; or Agent's endowment depends on some unobservable characteristics.

Adverse selection: A market process in which undesired results occur when one party has private information. Two ways to model adverse selection are to employ signaling games and screening games.

→ Our aim is *optimal contracting* when one party has private information:

2- Hidden Information

- **Screening:** *"Uninformed"* party moves first

    - ✓ It is screening because the uninformed party must attempt to screen the different pieces of information the informed party has.
    - ✓ E.g.: A lender usually knows less about the risk-return characteristics of a project than the borrower.
    - ✓ E.g.: Optimal income taxation: The tax policy maker has to design the rule based on unobservable variables like individuals earning, their potentials or productivity.

- **Signaling:** *"Informed"* party moves first

    - ✓ By "contract" we mean Institutional rule of contracting
    - ✓ It is signaling because the informed party signals her type.
    - ✓ E.g.: Education as a selection device => the person who knows about her qualities signaling her type by the education she has.
    - ✓ E.g.: Investment proposals and corporate financing

2-1- Screening

- In response to the problem of asymmetric information signaling develops in marketplace.

- Signaling models involve more sophisticated game theory arguments than screening models. The fact that in signaling the informed party moves first enriches the equilibrium outcomes because many conditional beliefs of the uninformed party may be self-fulfilling.

- Before signaling, we consider an alternative market response to the problem of unobservable productivity of agent (e.g. productivity of a worker or quality of a portfolio manager) in which the *uninformed* parties take steps to try to distinguish, or screen various types of individuals.

- This problem was first formally analyzed by Mirrless (1971) in the context of income taxation, Rotschild and Stiglitz (1976) and Wilson (1977) in insurance market.

2-1- Screening

- ***Players***
  - One worker and two employers

- ***Employers' payoff:***
  - The worker's productivity is either high (θ_H) or low (θ_L)
  - Employers decide on wage bids w_i, i = 1, 2. There is (perfect) Bertrand competition between them, i.e. the employer with the higher wage bid employs the worker.
  - The employers also choose working hours t_i , i = 1, 2.
  - To an employer, a worker of type θ with working hours t is worth: θ
  - If employer i employs the worker, her payoff is $\theta - w$; and zero otherwise (when $w < w_i$)

2-1- Screening

- We learned that after the two firms offered the contract *(w, t)*, depending on the type θ *high* or *low* of the worker the firm with the better offer for the worker would earn θ - *w*.

- Firms believe that the λ share of the workers are high:

$$Pr(\theta = \theta_H) = \lambda$$

- Type θ of the worker who earns the wage *w* has the utility:

$$U(w, t \,/\, \theta) = w - 0.5t^2 / \theta$$

- So the problem is:

$$\underset{w_H, w_L}{Max} \quad \lambda(\theta_H - w_H) + (1-\lambda)(\theta_L - w_L)$$

$$s.t. \quad w_H - \frac{t_H^2}{2\theta_H} \geq w_L - \frac{t_L^2}{2\theta_H} \quad (IC_H)$$

$$s.t. \quad w_L - \frac{t_L^2}{2\theta_L} \geq w_H - \frac{t_H^2}{2\theta_L} \quad (IC_L)$$

2-1- Screening

First best: Complete information

Proposition 13.D.1: In any SPNE of the screening game with observable worker types, a type θ_i worker accepts contract $(w_i^*, t_i^*) = (\theta_i, 0)$, and firms earn zero profits.

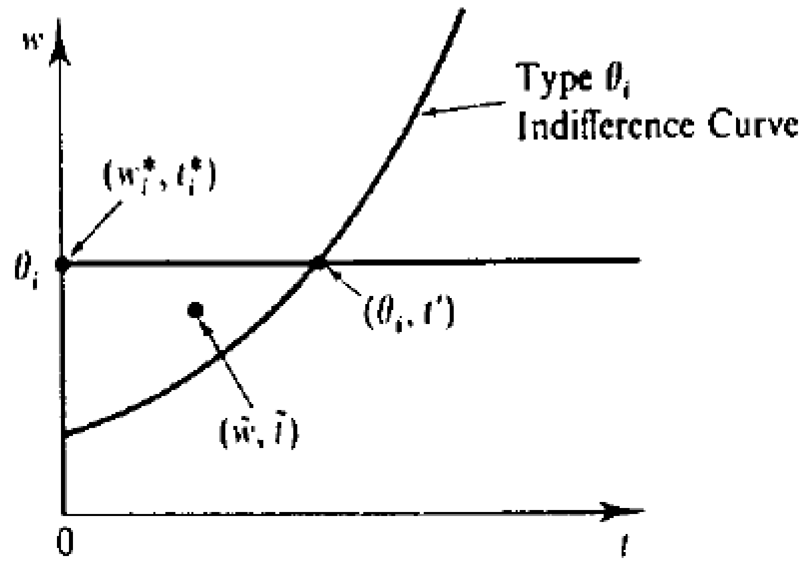
- ❑ Proof: by contradiction

Case 1: $w_i < \theta_i$

Firm j bids $w_i + \varepsilon$.

Case 2: $w_i > \theta_i$

Firm i looses.



- ❑ So $w_i = \theta_i$ and since t_i does not affect the outcome of the firm she would set $t_i = 0$ to attract the worker.

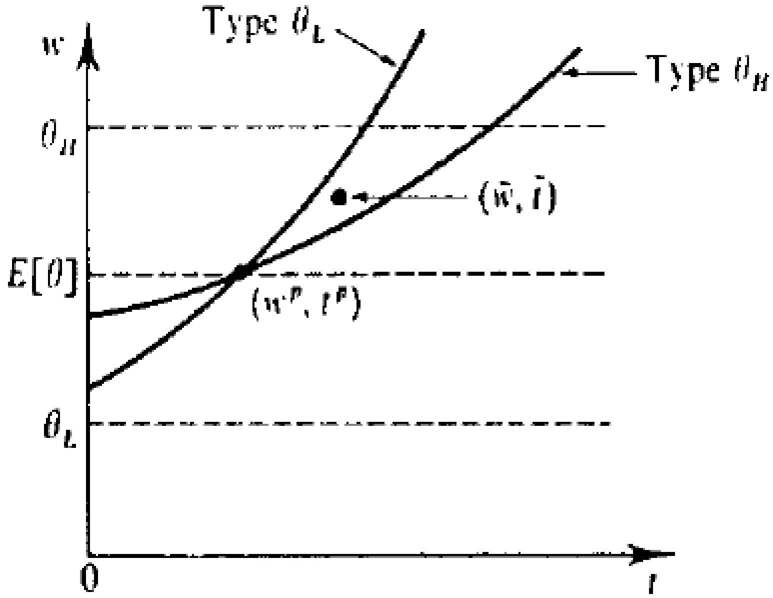
2-1- Screening

Pooling Equilibrium

- **Lemma:** No pooling equilibrium exists

- *Proof:* by contradiction

  Suppose (w, t) be the pooling equilibrium and firm i has employed the worker. However, firm j ≠ i may make a profit by proposing a different contract.



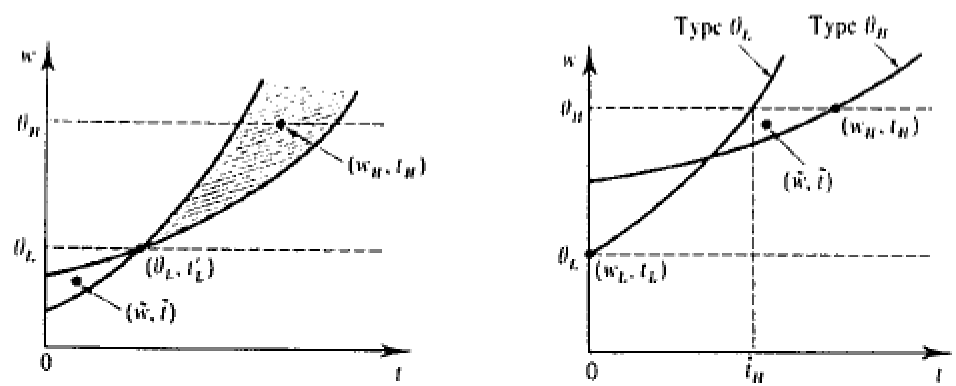
2-1- Screening

Separating Equilibrium

Proposition 13.D.2: In any subgame perfect Nash equilibrium of the screening game, low-ability workers accept contract $(\theta_L, 0)$, and high-ability workers accept contract $(\theta_H, \hat{t}_H)$, where $\hat{t}_H$ satisfies $\theta_H - c(\hat{t}_H, \theta_L) = \theta_L - c(0, \theta_L)$.

- *Proof:*

  $w_H = \theta_H$
  $w_L = \theta_L$
  $t_L = 0$



IC$_L$ and IC$_H$ both may not be binding. We have to show that IC$_L$ is binding because then

$$\theta_L - \frac{t_L^2}{2\theta_L} = \theta_H - \frac{t_H^2}{2\theta_L}$$

To show this consider IC$_H$ and IC$_L$ together and observe that they may not be binding together

2-1-1 Applications

Applications

2-1-1-1 Monopoly pricing

Adverse selection naturally arises from the following context first analyzed by Mussa and Rosen(1978). In this part we follow the notation by Bolton, Dewatripont pages 47-57.

Consider a transaction between a buyer and a seller in which the buyer's willingness to pay, $P(x,\theta)$, is not known to the seller:

$$\theta \begin{cases} \theta_H & \text{with probability } \beta \\ \theta_L & \text{with probability } 1-\beta \end{cases}$$

- For the demand x of type θ → Inverse demand curve is $P(x,\theta)$

$\Rightarrow$ Utility of the type θ agent from consuming q and transferring T is then:

$$u(q,T,\theta) = \int_0^q P(x,\theta)\,dx - T$$

2-1-1-1 Monopoly pricing

- However, For this part we shall consider the following special and convenient functional form

$$u(q,T,\theta)=\theta v(q)-T$$
$$s.t.\ \ v(0)=0,\ \ v'(q)>0,\ \ v''(q)<0$$

- Seller, however, decides on the pair *(T, q)* to maximize:

$$T-cq$$

- Therefore the problem is:

$$\underset{T,q}{Max}\ E_i\left(T_i-cq_i\right)$$
$$s.t.\ \ \theta_i v(q_i)-T_i\geq\bar{u} \qquad (IR)$$
$$s.t.\ \ q_i\in\arg\max_q \theta_i v(q)-T(q) \qquad (IC)$$

- ū could be endogenized, but here we shall treat it as exogenous and normalize it to zero.

2-1-1-1 Monopoly pricing

First Best (Known types): Perfect price discrimination

Types are known and no need to elicit their willingness to pay

$\Rightarrow$ Drop IC and bind IR because principal decides for everything

$$T_i = \theta_i v\left(q_i\right) \quad \& \quad \theta_i v'\left(q_i\right) = c$$

Intuitively, without adverse selection (with complete information) the seller finds it optimal to maximize total surplus by having the buyer select a quantity such that marginal utility equals marginal cost, and then setting the payment so as to appropriate the full surplus and leave no rent to the buyer above ū .

Without adverse selection, the total profit of the seller is thus:

$$Total \quad \pi_{Seller} = \beta\left(T_L - cq_L\right) + \left(1-\beta\right)\left(T_H - cq_H\right)$$

2-1-1-1 Monopoly pricing

Second Best: Incomplete information (Adverse selection)
Simple Contract: Linear Pricing i.e. T=Pq

Start with **Incentive Compatibility** $q_i \in \arg\max_q \theta_i v(q) - T(q)$

When types are not known the contract has to be incentive compatible, i.e. $q_i \in \arg\max_q \theta_i v(q) - Pq$, or:

$$\theta_i v'(q) = P$$

From this condition, derive the demand of type i → $q_i = D_i(P)$

And then compute the Buyer's surplus of type θ_i:

$$S_i(P) = \underbrace{\theta_i v(D_i(P))}_{\text{Utility of type } \theta_i} - \underbrace{PD_i(P)}_{\text{His payment}}$$

2-1-1-1 Monopoly pricing

Second Best: Incomplete information (Adverse selection)
Simple Contract: Linear Pricing i.e. T=Pq

Now solve **Seller's Perspective**:

Therefore, the Seller's perceived demand and surplus are:

$$D(P) = \beta D_L(P) + (1-\beta) D_H(P)$$

$$S(P) = \beta S_L(P) + (1-\beta) S_H(P)$$

The Seller's problem is to maximize T - cq = Pq - cq = (P – c)D(P), i.e.:

$$\underset{P}{Max}\,(P-c)D(P)$$

$\Rightarrow$ Monopoly price and welfare loss: $P_m = c - \dfrac{D(P)}{D'(P)} > c$

❖ Here the question is if the firm can do better than this?

2-1-1-1 Monopoly pricing

Second Best: Incomplete information (Adverse selection)
Contract: Two-part tariff i.e. T=Z+Pq

Z: Fixed fee (entry fee)
P: linear part (user fee)

If the monopoly decides to cover all the market she would set:

$$Z = S_L(P)$$

So the Seller's maximization changes to:

$$\underset{P}{Max}\ S_L(P) + (P - c)D(P)$$

$\Rightarrow$ Monopoly price under two-part tariff $P_d = c - \dfrac{D(P) + S_L{}'(P)}{D'(P)} > c$

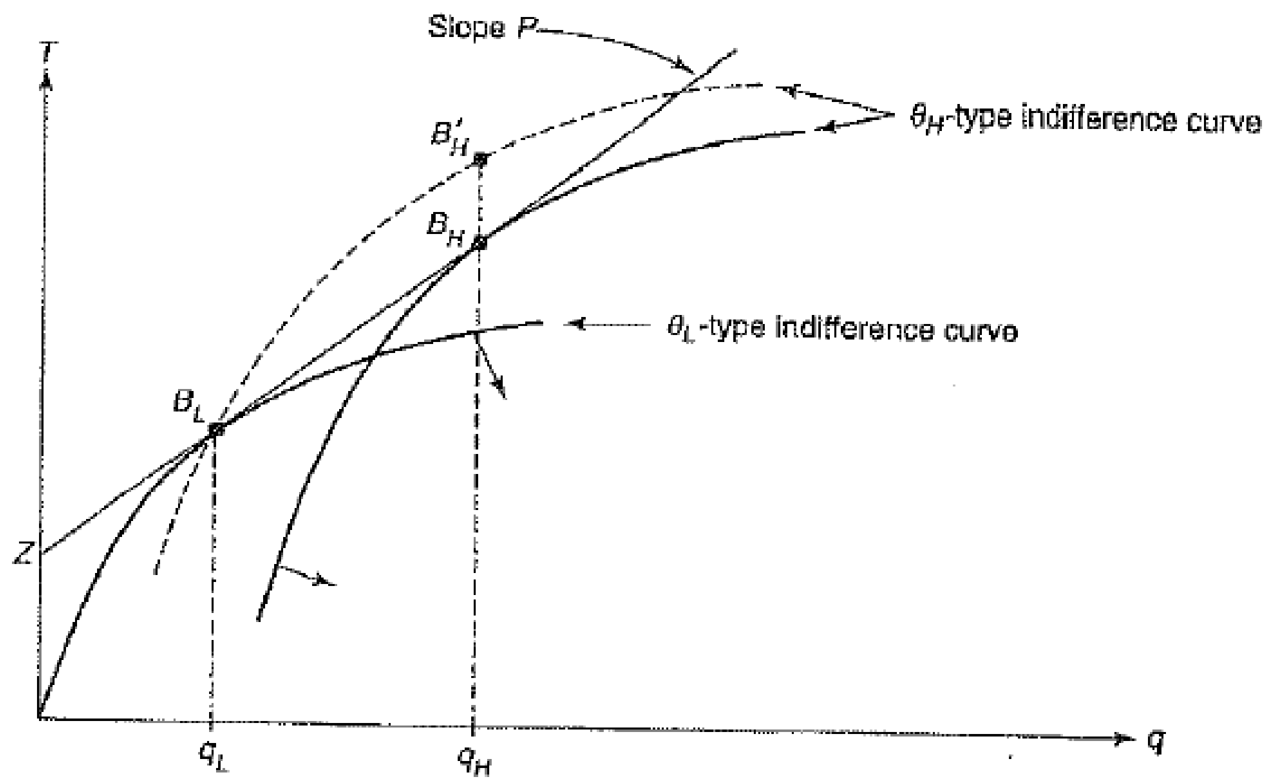
- ❖ Exercise: Show that $D(P) + S_L{}'(P) > 0$
- ❖ Exercise: Show that $P_m > P_d > P_c = c$

2-1-1-1 Monopoly pricing

Second Best: Incomplete information (Adverse selection)
Contract: Two-part tariff i.e. T=Z+Pq

An important feature of the optimal two-part tariff is that the θ_H-type buyer strictly prefers B'_H to B_L.

This suggests that there may be something in making the price "more nonlinear".



2-1-1-1 Monopoly pricing

Second Best: Incomplete information (Adverse selection)

Contract: Nonlinear Pricing (q, T(q))

So the problem is

$$\underset{q,\,T(q)}{Max}\ \beta\left(T(q_L)-cq_L\right)+(1-\beta)\left(T(q_H)-cq_H\right)$$

$$s.t.\quad q_i \in \arg\max_{q}\ \theta_i v(q)-T(q) \qquad (IC)$$

$$s.t.\quad \theta_i v(q_i)-T(q_i)\geq 0 \qquad (IR)$$

- Having two types → w.l.o.g restrict each schedule T(q) to the pair of optimal choices so define $T(q_i) = T_i$ => simplified ICs:

$$\theta_H v(q_H)-T_H \geq \theta_H v(q_L)-T_L \qquad (IC_H)$$

$$\theta_L v(q_L)-T_L \geq \theta_L v(q_H)-T_H \qquad (IC_L)$$

$$\theta_H v(q_H)-T_H \geq 0 \qquad (IR_H)$$

$$\theta_L v(q_L)-T_L \geq 0 \qquad (IR_L)$$

2-1-1-1 Monopoly pricing

Second Best: Incomplete information (Adverse selection)

Contract: Nonlinear Pricing (q, T(q))

Solution

Claim 1: IR_H is never binding

Proof: IR_L + IC_H + $\theta_H > \theta_L$ shows this

Claim 2: IC_L is never binding

Proof: type H may have the incentive to mimic type L because then he has to pay $T_L < T_H$ to earn $(\theta_H - \theta_L)v(q_L)$, but type L would never do so.

The problem now is:

$$\underset{q,\, T(q)}{Max}\ \beta\left(T_L - cq_L\right) + \left(1-\beta\right)\left(T_H - cq_H\right)$$

$$\theta_H v\left(q_H\right) - T_H \geq \theta_H v\left(q_L\right) - T_L \qquad (IC_H)$$

$$\theta_L v\left(q_L\right) - T_L \geq 0 \qquad (IR_L)$$

2-1-1-1 Monopoly pricing

Second Best: Incomplete information (Adverse selection)
Contract: Nonlinear Pricing (q, T(q))

Solution

Claim 3: IC_H is binding
Proof: Otherwise, increase T_H and objective improves.

Claim 4: IR_L is binding
Proof: Otherwise, increase T_L and objective improves.

❑ Substituting the binding constraint in the objective returns:

$$\underset{q_L,\, q_H}{Max}\ \beta\left(\theta_L v\left(q_L\right)-cq_L\right)+\left(1-\beta\right)\left(\theta_H v\left(q_H\right)+\theta_L v\left(q_L\right)-\theta_H v\left(q_L\right)-cq_H\right)$$

F.O.Cs:

$$\theta_H v'\left(q_H\right)=c \quad \theta_L v'\left(q_L\right)=\frac{c}{1-\left(\frac{\left(1-\beta\right)}{\beta}\frac{\theta_H-\theta_L}{\theta_L}\right)}>c$$

2-1-1-1 Monopoly pricing

Second Best: Incomplete information (Adverse selection)
Contract: Nonlinear Pricing (q, T(q))

Results

- ✓ $q^*_L < q^*_H$; $T'(q^*_L) > T'(q^*_H)$; $T(q^*_L) < T(q^*_H)$
  - ✓ θ_H → more interested in the good; pays marginally less; and transfers more to consume more
  - ✓ θ_L → less interested in the good; pays marginally more; and transfers less to consume less

- ✓ Efficiency at the top → the type θ_H buys as high as the first best

- ✓ The lowest type earns zero surplus (IR_L is binding) while the other type(s) obtain a strictly positive "informational" rent(s).
  - ✓ This result and the formers remain valid with more than two types

2-1-1-2 Income taxation

Income taxation

2-2- Signaling

- Signaling theory (ST) emerged from *economics* and *biology* in the mid-70s (Spence, Zahavi, Grafen)
  - and now is extremely widely applied in economics and other fields.
  - The seminal contribution by Nobel laureate Michael Spence (1973) is on education signaling; applications have been seen in entry deterrence through limit pricing, advertising, monetary policy, legal bargaining and …

❖ Loosely speaking, signaling is the costly undertaking of actions in order to either convey or hide private information from others

2-2- Signaling

- Signaling tackles a fundamental problem of communication:
  - How can an agent, **the receiver**, establish whether another agent, **the signaler, is telling or otherwise conveying the truth** about a state of affairs or event which the signaler might have an interest to misrepresent?
  - And, conversely, how can **the signaler persuade the receiver that he is telling the truth**, whether he is telling it or not?

- This two pronged questions potentially arise every time the *interests* between signalers and receivers diverge or collide and there is *asymmetric information*.

2-2- Signaling

Signaling models usually involve a "Sender" and a "Receiver" and include the following steps:

1. Nature draws a type θ_i *in* Θ_i for the Sender

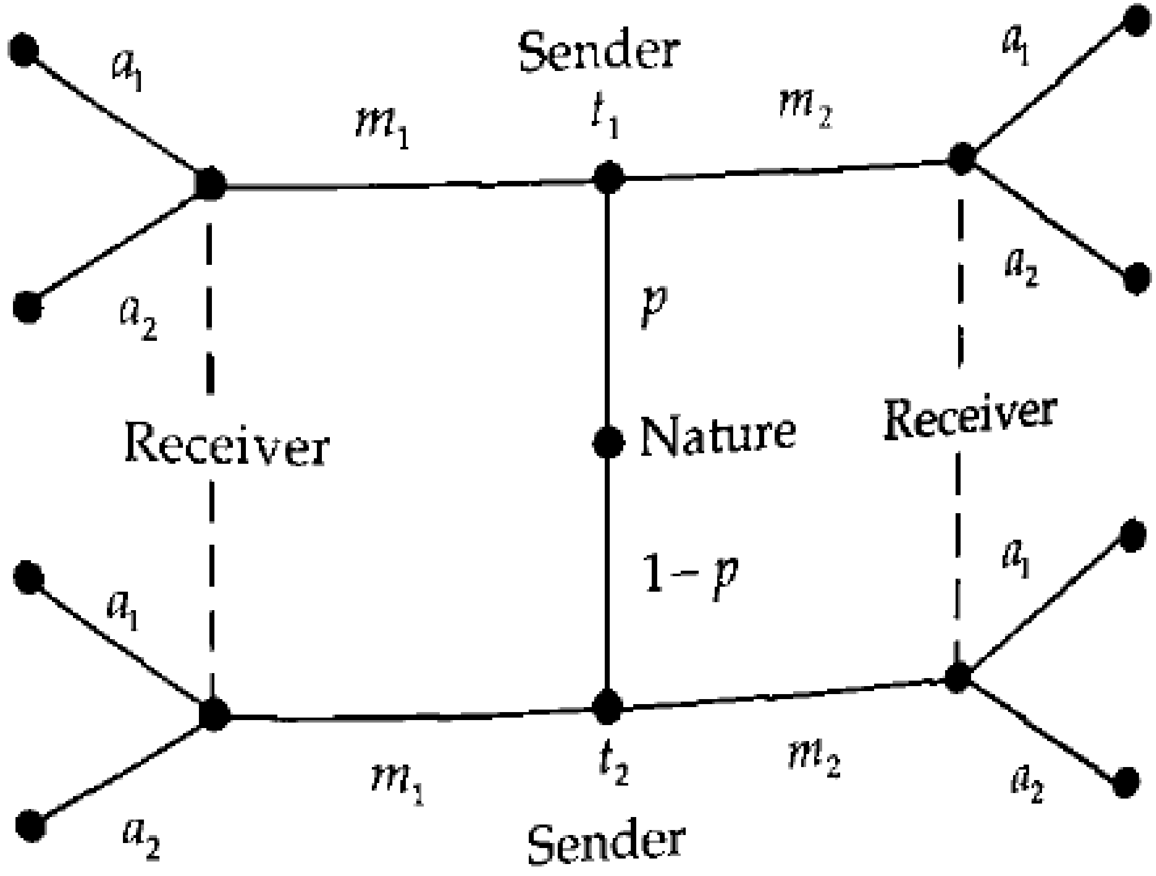
2. The Sender observes θ_i and then chooses a message m_j from $M = \{m_1 , m_2, \ldots, m_J\}$

3. The Receiver observes m_j (but not θ_i) and then chooses an action a_k from $A = \{a_1 , a_2, \ldots, a_K\}$

4. Payoffs are given by $U_S(\theta_i, m_j, a_k)$ and $U_R(\theta_i, m_j, a_k)$

2-2- Signaling

Another form of showing signaling games:



2-2- Signaling

- As mentioned before a *central issue in signaling games* is what kind of messages the Sender will choose *strategically* in order to hide—or reveal—information from the Receiver

✓ E.g.: An efficient incumbent firm would like to send a (price) message to a potential entrant along the lines "look, I am efficient; you better stay out of the market"

o problem: obviously an **inefficient firm would like to send the same message**, even though with complete information it would not be able to prevent entry

- How can the efficient incumbent credibly signal his type?

2-2-1 The Spence education model

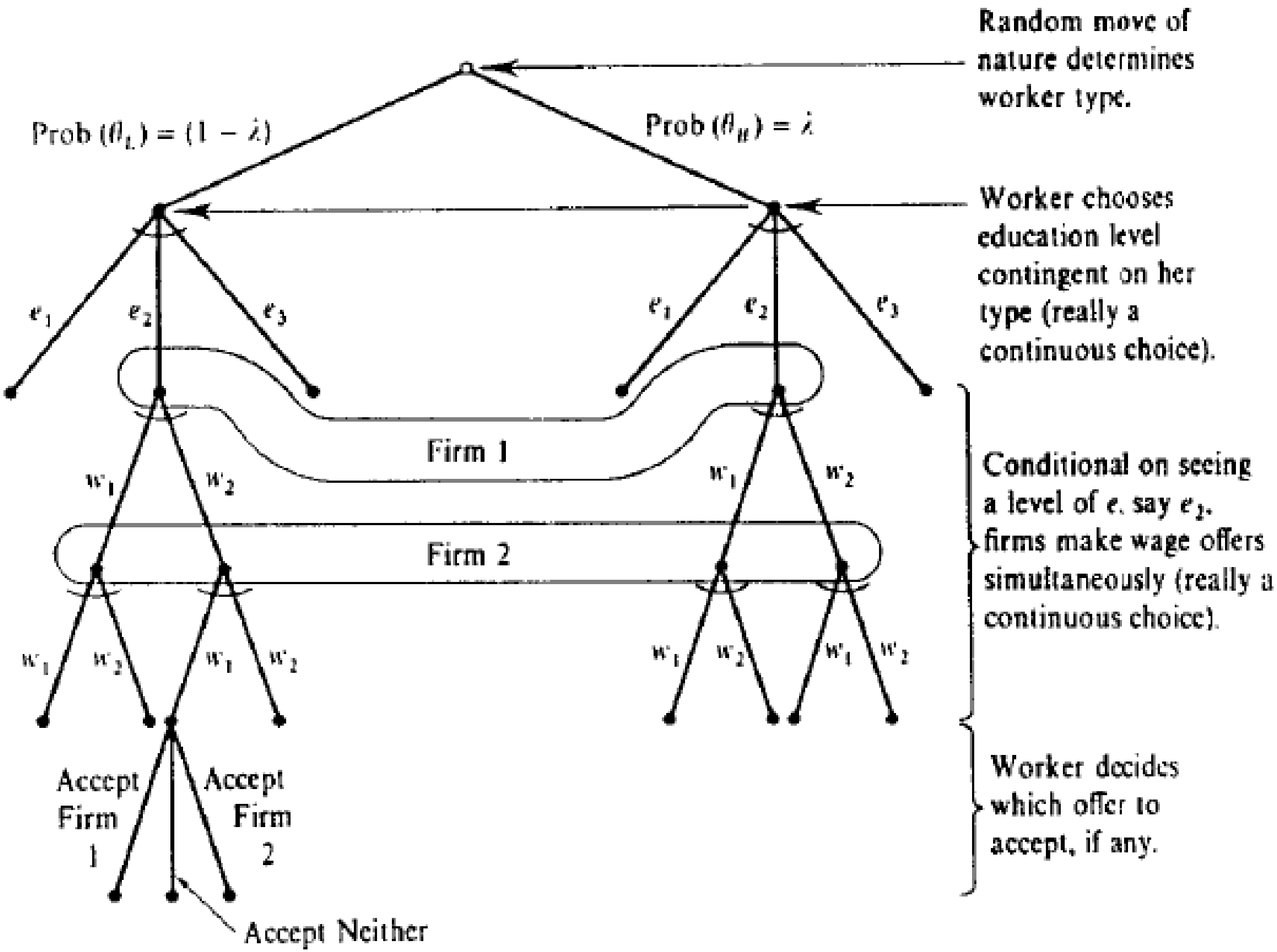


Figure 13.C.1
The extensive form of the education signaling game.

2-2-1 The Spence education model

- Employers in the labour market often know only *imperfectly* how capable potential employees (workers) are

- Spence's idea is that education *(say, the number of years at school and university, or the completion of a degree)* can serve as a signal for productivity

- We assume that there are only two types of players, **high** and **low** productivity workers. Both can obtain education but, crucially, **for high types this is easier** (it takes them less effort, time, or money)

- Under which conditions can the high type signal her superior capability through her education choice?

2-2-1 The Spence education model

- Three players: one worker and two employers
- The worker's productivity is either high ($\theta_H = 1.5$) or low ($\theta_L = 1$)
- Ex-ante chances that the worker is high ($pr(\theta=1.5)$) are λ
- The worker chooses education e from $[0, \infty)$, and the cost for this is
$$\frac{e^2}{2\theta}$$
- Seeing the education choice but not knowing the type of the worker, the employers update their beliefs using Bayes' Rule, where p denotes the posterior belief

2-2-1 The Spence education model

- Employers decide on their wage bids w_i, i = 1, 2. There is (perfect) Bertrand competition between them

  To an employer, a worker of type θ with education e is worth

  $$\theta \times e$$

  The employer with the higher wage bid employs the worker

- The worker's payoff is:

  $$w - \frac{e^2}{2\theta}$$

- If employer i employs the worker, her payoff is

  $$\theta e - w$$

  and zero otherwise (when $w < w_i$)

2-2-1 The Spence education model

- To be in line with our principal-agent notation we may write this problem as:

$$\underset{w_H, w_L}{Max} \quad \lambda(\theta_H e_H - w_H) + (1-\lambda)(\theta_L e_L - w_L)$$

$$s.t. \quad w_H - \frac{e_H^2}{2\theta_H} \geq w_L - \frac{e_L^2}{2\theta_H} \qquad (IC_H)$$

$$s.t. \quad w_L - \frac{e_L^2}{2\theta_L} \geq w_H - \frac{e_H^2}{2\theta_L} \qquad (IC_L)$$

- Note that we must not underestimate this problem and transfer it to a Lagrangian because:
  - There are two firms playing a Bertrand
  - Sufficient conditions of optimality are not satisfied
  - The problem of multiple equilibria that we will discuss later

2-2-1 The Spence education model

First best solution (Complete information)

- Bertrand competition => When θ is known, employers will simply bid as much as the worker is worth to them, that is, they bid θe

- Then a worker of type θ knows that, by bidding e, she can obtain $w = \theta e$ from the employers. Thus, the worker will choose e to maximize

$$\theta e - \frac{e^2}{2\theta}$$

$\Rightarrow$ The optimal education level is $e=\theta^2$

$\Rightarrow$ The type $\theta = 1$ chooses $e = 1$ and $\theta = 1.5$ chooses $e = 2.25$

$\Rightarrow$ Type θ earns $w = \theta e = \theta^3$ and has a payoff of $\pi = \theta e - e^2/2\theta = \theta^3 - \theta^4/2\theta = \theta^3/2$

$\Rightarrow$ The type $\theta = 1$'s profit is 0.5 and $\theta = 1.5$'s profit is 1.6875

2-2-1 The Spence education model

First best solution (Complete information)

- The optimal education level is thus $e^{*}=\theta^{2}$, such that $\theta=1$ chooses $e=1$ and $\theta=2$ chooses $e=4$

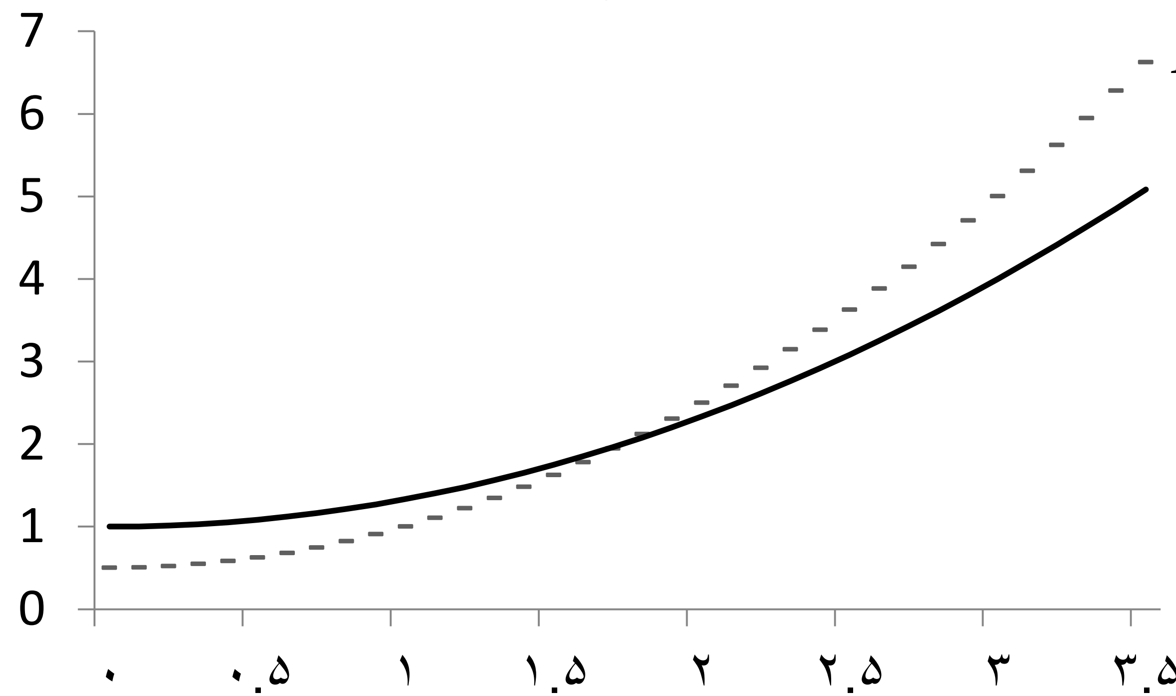
- That is, low would get $w=1$ and high would get a wage of $w = (1.5)^3 = 3.375$. Low's payoff is 0.5, high's is 1.6875 with complete information (i.e., known θ)

- Importantly, the complete information solution is not incentive compatible with incomplete information. We now show that low has an incentive to choose high's education choice $e = 2.25$ when θ is unknown
  - Choosing e = 2.25 yields a wage of 3.375
  - Thus choosing e = 2.25 would give the low type a higher payoff than her complete information education

$$3.375 – 2.252/2·1 = 0.84375 > 0.5$$

2-2-1 The Spence education model

- Workers earn a payoff of π = $w - e^2/2\theta$
- Thus, an *iso-payoff curve follows* $w = \pi + e^2/2\theta$ *(telling* us which wage-education combinations yields the same payoff π). We plot the curves for θ*=1 and* θ*=1.5* (the payoffs π = 0.5 and π = 1 are arbitrary examples)



$I_{\theta=1} = 1/2 + e^2/2$

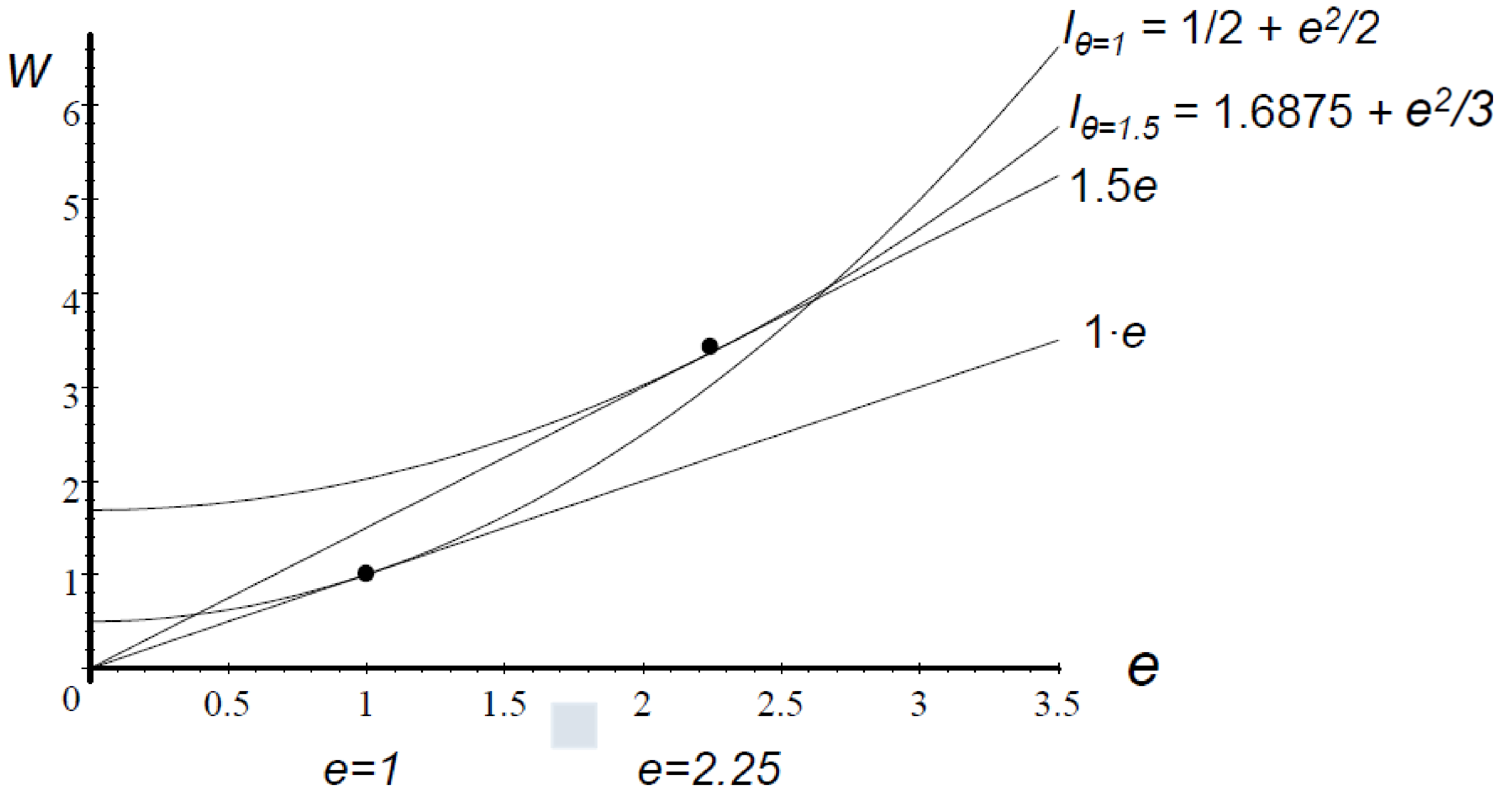
$I_{\theta=1.5} = 1 + e^2/3$

➢We can see that the curve for θ*=1* is always steeper.

➢The two curves intersect once at most (call it *Single-Crossing Property* or *SCP*)

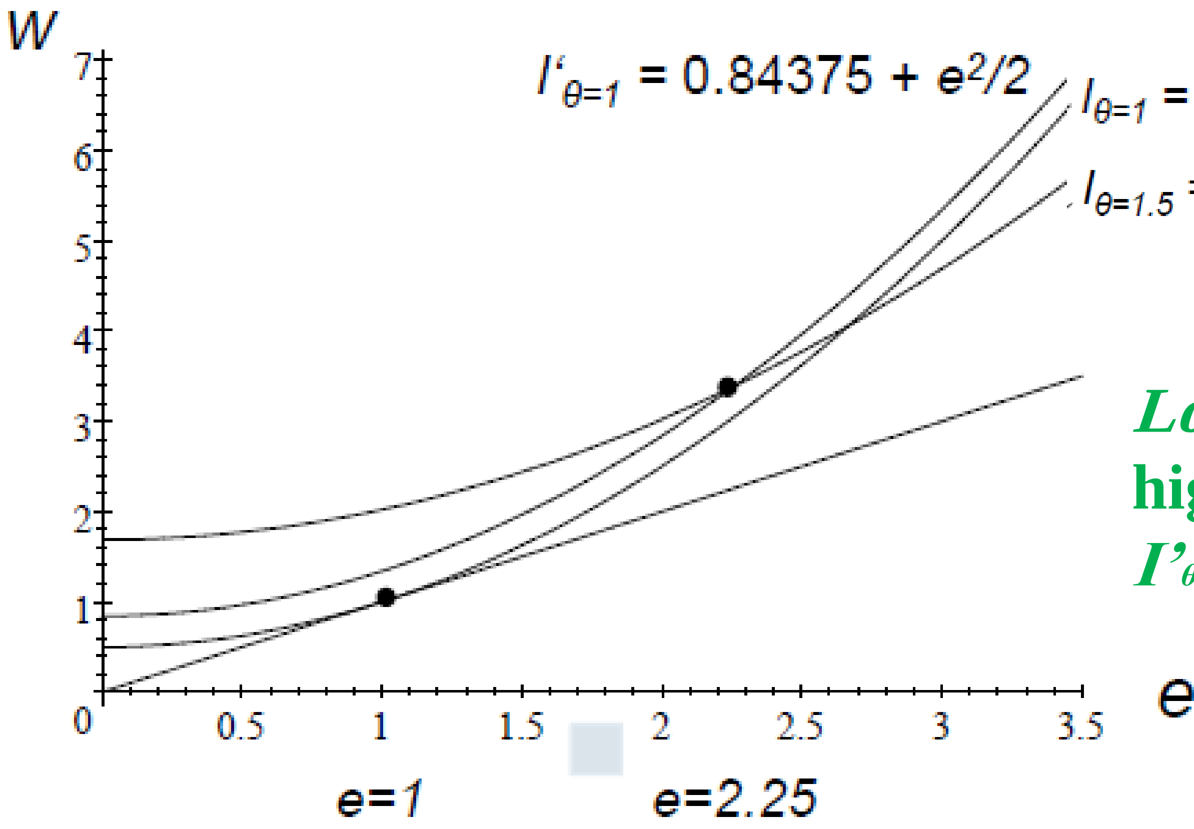
2-2-1 The Spence education model

With complete information, *θ is known and employers* bid $w=\theta e$. Plotting both θe and the iso-profit curves yields the optimal choices under complete information: at the tangency points, the highest payoffs occur



2-2-1 The Spence education model

We can see from the graphical analysis that the complete information optimal choices are not incentive compatible



Low can move up to a **higher iso-payoff curve** ***$I'_{\theta=1}$ by choosing e=2.25***

2-2-1 The Spence education model

The incentive compatibility constraints (if $e^*_H \neq e^*_L$)

- The lesson: actions have to be *incentive compatible in order to be part of an* equilibrium: *low must not find it profitable to mimic high* and vice versa

- We now search for such incentive compatible equilibrium actions. Let $\mathbf{e}^*_L$ *and* $\mathbf{e}^*_H$ *denote possible* equilibrium actions for *high and low, respectively*

- **Assumption:** *Suppose that* $e^*_H \neq e^*_L$ *in equilibrium*

- With $e^*_H \neq e^*_L$ *, wage bids will be w=e and w=1.5e after* the employers observe e^*_L *and* e^*_H*, respectively*

2-2-1 The Spence education model

The incentive compatibility constraints (if $e^*_H \neq e^*_L$)

- Thus, the incentive compatibility constraints read

$$1.5 \times e_H^* - \frac{e_H^{*\,2}}{2\theta_H} \geq 1 \times e_L^* - \frac{e_L^{*\,2}}{2\theta_H} \quad (IC_H)$$

$$1 \times e_L^* - \frac{e_L^{*\,2}}{2\theta_L} \geq 1.5 \times e_H^* - \frac{e_H^{*\,2}}{2\theta_L} \quad (IC_L)$$

- IC_L ensures that low does not want to imitate high's action, and IC_H prevents high from mimicking low

- There are many solutions to this system of inequalities. We can make a step ahead, though, by noting that *low* will choose *$e^*_L = 1$ in an PBE with $e^*_H \neq e^*_L$*

- Why? After observing $e^*_L < e^*_H$, *the employers know for sure* that the worker is *low and will thus bid $1 \cdot e^*_L$*

2-2-1 The Spence education model

The incentive compatibility constraints (if $\mathbf{e}^*_H \neq \mathbf{e}^*_L$)

- But from the complete-information analysis above, we know already that $\mathbf{e}^*_L = 1$ maximizes low's payoff in this case. That is, given she is identified as low, the worker cannot do better than to choose e = 1. Thus, $\mathbf{e}^*_L = 1$.

- Now we can solve for $\mathbf{e}^*_H$. With $\mathbf{e}^*_L = 1$, IC_L holds when:

$$\mathbf{e}^*_H \geq 2.618$$

- For $\mathbf{e}^*_L = 1$, IC_H becomes $1.5 \cdot \mathbf{e}^*_H - (\mathbf{e}^*_H)^2/(3) \geq 2/3$. Taking the relevant root, we obtain:

$$\mathbf{e}^*_H \leq 4.1375$$

2-2-1 The Spence education model

The incentive compatibility constraints (if $e^*_H \neq e^*_L$)

- Taking the two inequalities together, we have

$$4.1375 \geq \mathbf{e}^*_H \geq 2.618$$

where, to begin with, we will focus on $\mathbf{e}^*_H = 2.618$

- It appears we have found a solution (at least for the case where $\mathbf{e}^*_L \neq \mathbf{e}^*_H$)

- Note that $\mathbf{e}^*_H = 2.618 \geq 2.25$. That is, the lowest incentive compatible education level for high is higher than the complete-information education level

- This increase in education is called a ***signaling distortion***. In order to convince employers that the worker is high, a higher level of education has to be chosen than under complete information

- That is, *signaling is inefficient here.* Inefficiencies regularly occur in games of incomplete information

2-2-1 The Spence education model

Separating PBE تعادل بیزی کامل جداکننده

- An equilibrium with $\mathbf{e}^*_L \neq \mathbf{e}^*_H$ is called a *separating equilibrium*, or *separating PBE*

- It remains to check whether this separating outcome actually meets the requirements of PBE

➢ The equilibrium candidate has $\mathbf{e}^*_L = 1$ and $\mathbf{e}^*_H = 2.618$

✓ PBE 1st requirement: Specify beliefs

$$p = pr(\theta=1.5 \,/\, e = \mathbf{e}^*_L) = 0$$

and

$$p = pr(\theta=1.5 \,/\, e = \mathbf{e}^*_H) = 1$$

✓ PBE 2nd requirement: sequential rationality → checked through ICs

2-2-1 The Spence education model

Separating PBE

- ✓ PBE 3rd requirement: consistency with Bayes' Rule (this will be trivial but it is useful to go through the exercise):

$$\Pr(\theta = 1.5 \mid e = 1) = \frac{\Pr(e = 1 \mid \theta = 1.5)\lambda}{\Pr(e = 1 \mid \theta = 1.5)\lambda + \Pr(e = 1 \mid \theta = 1)\left(1 - \lambda\right)} = \frac{0}{1 - \lambda} = 0$$

$$\Pr(\theta = 1.5 \mid e = 2.618) = \frac{\Pr(e = 2.618 \mid \theta = 1.5)\lambda}{\Pr(e = 2.618 \mid \theta = 1.5)\lambda + \Pr(e = 2.618 \mid \theta = 1)\left(1 - \lambda\right)} = 1$$

$\Rightarrow$ Thus, beliefs on the equilibrium path are consistent with Bayes' Rule

- ✓ Consistency of out-of-equilibrium beliefs
  - ✓ For these cases, let us specify beliefs like the employers believe it was low who may send any of the out-of-equilibrium messages

2-2-1 The Spence education model

Separating PBE

- From the ICs above, we already know that neither type of the sender would want to deviate to any $e \neq e^*_L$, $\mathbf{e}^*_H$:

- High is better off choosing the equilibrium action as just seen
- Low is better off choosing e=1, as this maximizes her payoff when she is believed to be low

$\Rightarrow$ Thus neither type will choose any $e \neq e^*_L$, $\mathbf{e}^*_H$:

- Another deduction:
  - $e^*_L < \mathbf{e}^*_H$ => Employers know the type L and she is better-off by $e^*_L = 1$
  - $e^*_L > \mathbf{e}^*_H$ => It is against IC_L that L wouldn't mimic H

2-2-1 The Spence education model

Separating PBE

We can now state a complete separating PBE:

- Worker types choose $\boldsymbol{e}^*_L = 1$ *and* $\boldsymbol{e}^*_H = 2.618$
- Employers' beliefs are

$$p = prob.(\theta = 1.5 \mid e = \boldsymbol{e}^*_H) = 1$$

and

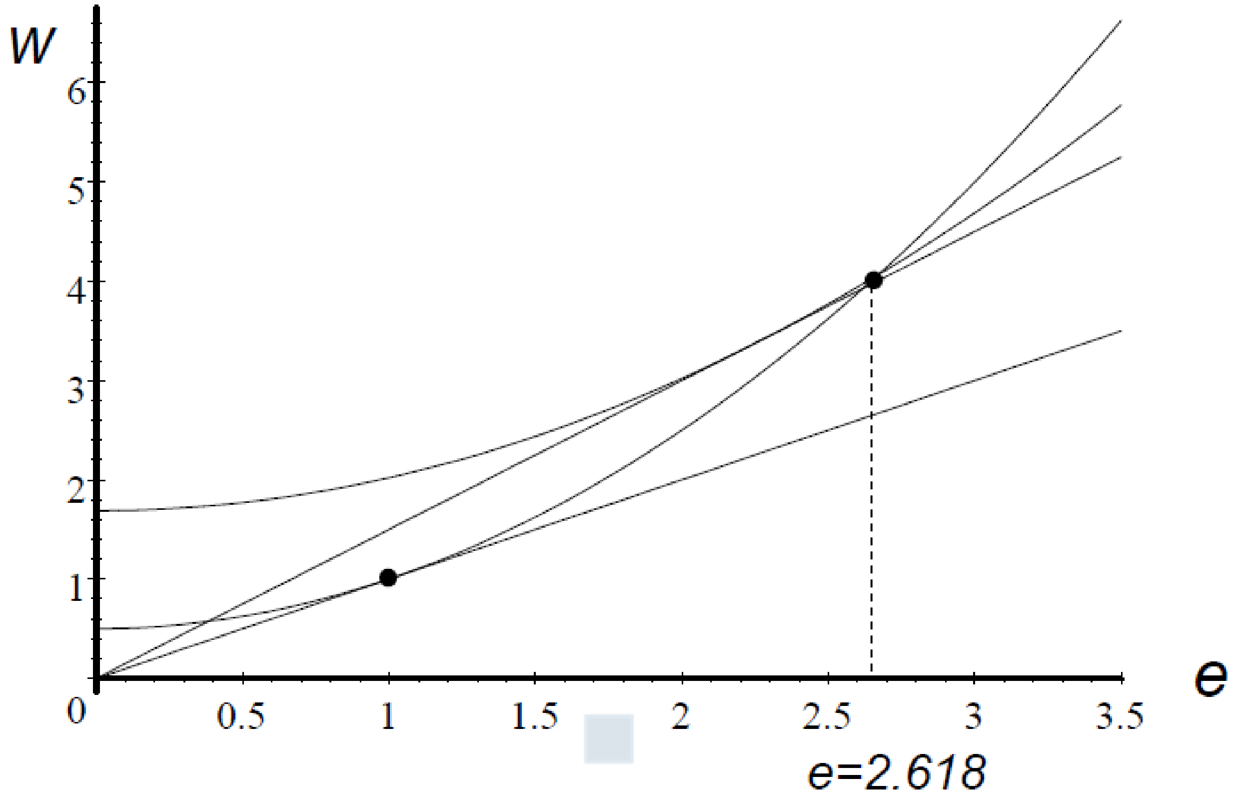
$$p = prob.(\theta = 1.5 \mid e \neq \boldsymbol{e}^*_H) = 0$$

- Employers bid $w = 3.927$ after observing $\boldsymbol{e}^*_H = 2.618$ and $w = \mathbf{e}^*_L$ otherwise.

2-2-1 The Spence education model

Separating PBE

- By choosing $\boldsymbol{e^*_H} = 2.618$, high ensures that low cannot mimic her. We say that high separates herself from low



2-2-1 The Spence education model

Separating PBE

- Unfortunately, there are further separating PBE
- We saw that (given $e^*_L = 1$) any education level that meets $2.618 \leq e^*_H \leq 4.1375$ *would work*
- We focused on the so-called *least-cost separating* equilibrium where $e^*_H = 2.618$
- It is common to do this (and below we will deliver an argument in favour of this PBE) but, again, any $2.618 < e^*_H \leq 4.1375$ can also be a separating PBE
- We will come back to the problem of multiple equilibria

2-2-1 The Spence education model

Pooling PBE تعادل بیزی کامل استخری

- The entire previous analysis was based on the assumption that $\boldsymbol{e^*_L \neq e^*_H}$ *in equilibrium*

- There is, however, also the possibility that $\boldsymbol{e^*_L = e^*_H}$. This case is called ***pooling behavior***

- **Assumption:** *Suppose that* $\boldsymbol{e^*_L = e^*_H}$ *in equilibrium*

- In a pooling equilibrium, Receivers (employers) do not learn anything from the message they observe

- After observing $\boldsymbol{e^* = e^*_L = e^*_H}$, *the posterior belief (پس انگاره)* must be equal to *the prior belief*

$$\Pr(\theta = 1.5 \,|\, e = e^*) = \frac{\Pr(e = e^* \,|\, \theta = 1.5)\lambda}{\Pr(e = e^* \,|\, \theta = 1.5)\lambda + \Pr(e = e^* \,|\, \theta = 1)\left(1-\lambda\right)} = \lambda$$

2-2-1 The Spence education model

Pooling PBE

- We need to specify beliefs for $e \neq e^*$ and, again, Bayes' Rule does not pin down these beliefs. We specify

$$\Pr(\theta = 1.5 \,|\, e \neq e^*) = 0$$

- The employers believe it was low who sent any of the out-of-equilibrium messages

- We can now check which realizations of $\boldsymbol{e}^* = \boldsymbol{e}^*_L = \boldsymbol{e}^*_H$ are incentive compatible

- Employers holding the posterior belief of p = λ will bid the expected productivity of the worker $\lambda(1.5\boldsymbol{e}^*) + (1-\lambda)\,\boldsymbol{e}^*$. That is, by choosing $\boldsymbol{e}^*$, either type of the worker can ensure herself a wage of $(1 + \lambda/2)\,\boldsymbol{e}^*$

2-2-1 The Spence education model

Pooling PBE

- **Out-of-equilibrium belief:** When defecting to some $e \neq e^*$, the worker is believed to be low, and will thus obtain a wage of $1 \cdot e$, therefore:
- Defection's payoff: $\begin{cases} e - \dfrac{e^2}{2} & \text{for low} \\ e - \dfrac{e^2}{3} & \text{for high} \end{cases}$
- The maximum payoff from $e \neq e^*$ occurs for $e = 1$ for low and $e = 3/2$ for high leading to the maximum possible defection payoffs 0.5 and 0.75 for low and high, respectively.
- We saw that choosing $\boldsymbol{e}^*$ yields a wage of $(1 + \lambda/2)e^*$, so equilibrium payoffs are: $(1+\lambda/2)e^* - (e^*)^2/2\theta$

2-2-1 The Spence education model

Pooling PBE

- When is the equilibrium payoff, $(1 + \lambda /2)e^* - (e^*)^2/2\theta$, larger than the defection payoff of 0.5 and 0.75? We illustrate the solution for $\lambda = 0.5$

- For low, we must have $(1 + 1/4)e^* - (e^*)^2/2 \geq 0.5$, which turns:

$$0.5 \leq e^* \leq 2$$

- For high, we must have $(1 + 1/4)e^* - (e^*)^2/3 \geq 0.75$, *which is*

$$0.75 \leq e^* \leq 3$$

- Thus, as long as $0.75 \leq e^* \leq 2$, both types prefer to adhere to the equilibrium education e^* (if $\lambda = 0.5$)

2-2-1 The Spence education model

Pooling PBE

- We now state the complete pooling PBE for $\lambda = 0.5$

- Worker types choose $\boldsymbol{e}^*_L = \boldsymbol{e}^*_H = e^*$, *where* $0.75 \leq e^* \leq 2$

- Employer beliefs are

$$p = pr(\theta{=}1.5 \,/\, e = e^*) = \lambda, \text{ and}$$
$$p = pr(\theta{=}1.5 \,/\, e \neq e^*) = 0$$

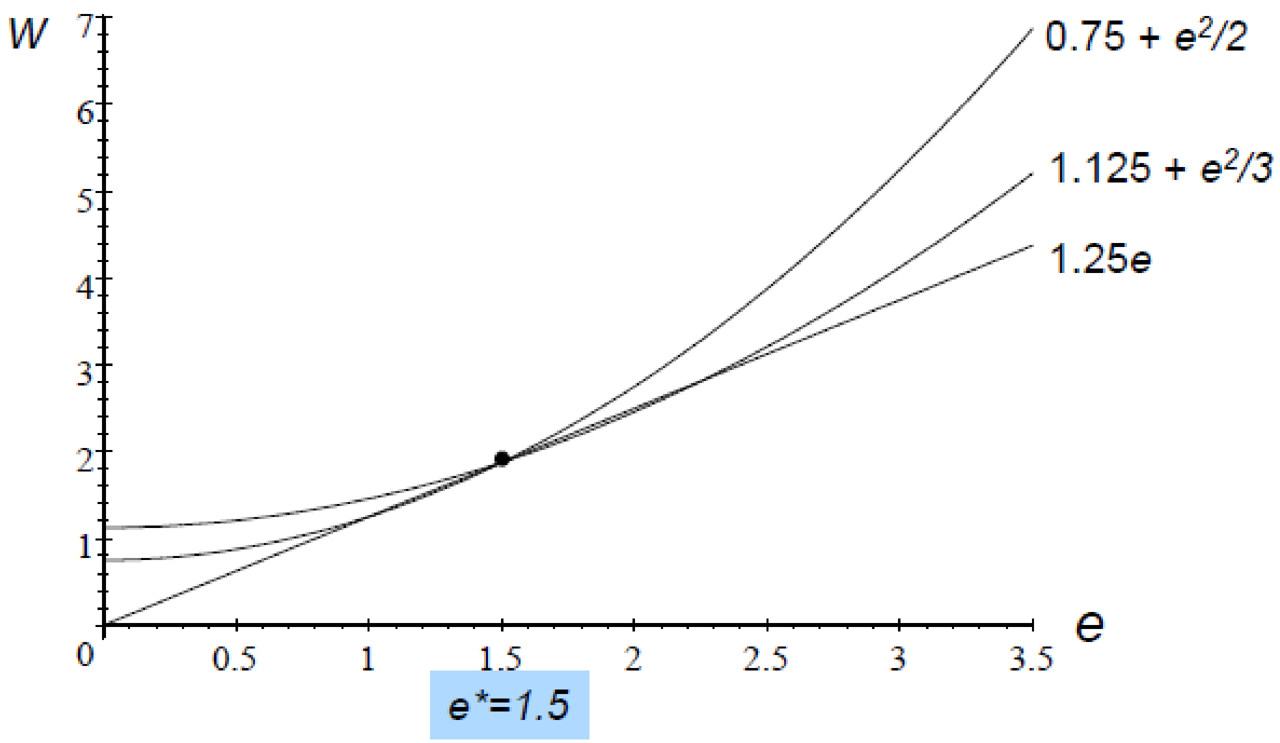
- Employers bid $w = (1 + \lambda/2)e^*$ after observing e^* and w=1e otherwise

❑ Our incentive compatibility analysis ensures that the sequential rationality requirement is met. This completes the analysis.

2-2-1 The Spence education model

Pooling PBE

The example assumes $e^* = 1.5$ and $\lambda = 0.5$. The two iso-payoff curves intersect at $e^*=1.5$, where the worker gets w = 1.25 e^* = 1.875. Choosing some other e, would push them down to the wage line 1e



2-2-1 The Spence education model

- The PBE analysis of the Spence game has shown that there are both separating and pooling equilibria

- Specifically, it turned out there is a continuum of separating equilibria (for $2.618 \leq \boldsymbol{e}^{*}_{\boldsymbol{H}} \leq 4.1375$) and also a continuum of pooling equilibria (for $\lambda=0.5$: $0.75 \leq e^{*} \leq 2$)

- There may, in addition, be so-called *hybrid equilibria* where one type chooses some education with probability 1 and the other type randomizes between pooling with and separating herself from the first type (see Gibbons, 1992, p. 202-205)

2-2-1 The Spence education model

Refinements

- An odd point in the analysis of the PBE was that we were free to fix beliefs for an out-of-equilibrium event

- For both the derivation of the separating and the pooling equilibria, we chose beliefs such that probability 1 is on the *low type selecting this action*

- But is this reasonable? Why should it be the *low type* for sure that deviates, regardless of the level of education? Would it not be more plausible to check whether some type could profitably send such an out-of-equilibrium message under some conditions?

- This is indeed the agenda of the so-called *belief-based refinements like the Intuitive Criterion (معیار شهودی)*

2-2-1 The Spence education model

Intuitive Criterion

- Here, we will learn (only briefly) about one refinement criteria which is useful in signaling: the Intuitive Criterion (Cho and Kreps, 1987)
- Consider again the pooling equilibrium with λ=0.5 and e* = 1.5, and suppose the employers observe an out-of-equilibrium education level e' = 2.75
- The belief we fixed to support the initial equilibrium is p = prob.(θ=1.5 | e $\neq$ 1.5) = 0, that is, low is believed to be the deviator
- However, low could never benefit from such a deviation. The highest possible wage with e' = 2.75 is 1.5·2.75, then low type would earn $1.5\times2.75 - (2.75)^2/2 = 0.344$
- But low can always secure itself a payoff of 0.5 by choosing e = 1

2-2-1 The Spence education model

Intuitive Criterion

- However, if the out-of-equilibrium education level e’ = 2.75 is unlikely to be chosen by low, the Receivers (employers) should assign no positive weight on low and should believe $p = pr(\theta=1.5 \mid e = 2.75) = 1$

- But with $pr\ (\theta=1.5 \mid e=2.75) = 1$, the high type has a profitable deviation $1.5\times2.75 - (2.75)^2/3 = 1.604$ which is more than the equilibrium payoff of 1.125

- This kind of argument destroys the pooling equilibrium

- Indeed, one can show that such arguments eliminate all equilibria (pooling and separating) except for the least-cost separating equilibrium we analyzed

2-2-1 The Spence education model

Intuitive Criterion

- Even though the intuition should be clear, we have to state the point more formally. Two different notions of *dominance can be applied here*

Dominance: A *message m_j is dominated for type θ_i if there exists another message m'_j such that the lowest possible payoff from sending m'_j is higher than the highest possible payoff from sending m_j . If message m_j is dominated for type θ_i, the Receiver should assign zero probability on type θ_i sending the message m_j.*

- This is the argument we applied above. We first derived low's highest possible payoff from sending e'=2.75 and then compared it to the highest possible payoff from another message (e=1) which turned out to be higher

2-2-1 The Spence education model

Intuitive Criterion

- Here is an even more forceful concept

Equilibrium Dominance: A *message* m_j *is equilibrium dominated for type* θ_i *if* θ_i*'s equilibrium payoff is greater than the highest possible payoff from sending* m_j.

Intuitive criterion: If a message m_j is equilibrium dominated for type θ_i, the Receiver should assign zero probability on type θ_i

(A problem arises if the message is equilibrium dominated for all types in the set of types)

2-2-1 The Spence education model

Intuitive Criterion

- To see the additional power from Equilibrium Dominance, take again the pooling equilibrium with λ=0.5, e*=1.5 and payoffs of 0.75 and 1.125
- Consider now a deviation with e'=2.5. The highest possible payoff from the message for low is now

$$1.5 \cdot 2.5 - (2.5)^2/2 = 0.625$$

- By choosing e = 1, the lowest possible payoff low can ensure herself is 0.5—which is smaller than 0.625, and thus low could have sent that message
- However, low's highest possible payoff from sending e'=2.5 is still lower that its equilibrium payoff of 0.75 (with e* = 1.5). Thus e' =2.5 is equilibrium dominated for low
- If Receivers (employers) follow the Equilibrium Dominance argument and assign no positive weight on low accordingly, then high type has a profitable deviation again. This breaks the pooling equilibrium

2-2-1 The Spence education model

Intuitive Criterion

A final remark on refinements

- There are many more such equilibrium refinements, and the quest for the ultimate refinement has been mocked as the "Search for the Holy Grail" of game theory

- Still, the problem of multiple equilibria is severe and equilibrium refinements are essential and useful in tackling this problem

3- Findings, Potentials and restrictions of Contract theory

Findings (Revelation principles) قواعد آشکارسازی

Guarantees and *minimum standards* , and *mandatory contract cancellation permits* solve the problem of market for lemons → Lemons laws in automobile market require automakers to buy back cars that experience frequent problems in the first year of use.

With nonlinear pricing we implement efficiency at top so at least for high types the first best outcome is found.

In moral hazard having *the same risk aversion* principal can implement the optimum effort from agent. (Though it is not called a revelation principal here)

3- Findings, Potentials and restrictions of Contract theory

Restrictions

✓ *Holmstrom Milgrom (1991)* in an interesting setup show that when one dimension of effort is not observable it is better to relax wage scheme dependence on the observable dimension

⇒ It is better to relax the dependence between teachers' evaluations or researchers' working hours and their payments.

➢ Problem: No incentive to work!

❑ One solution may be manipulating on their career concerns

3- Findings, Potentials and restrictions of Contract theory

Restrictions

Indeed, in a major result which builds on ***Arrow's*** **impossibility theorem**, Gibbard (1973) and Satterthwaite (1975) have shown that it is impossible in general to attain the full information efficient outcome when:

- there are more than two (≥3) possible allocations to choose from.
- And, the set of possible types of each contracting parties' domain is very diverse. (i.e. domain of preferences is unrestricted)

Gibbard-Satterthwaite theorem, in fact, shows that if such a principle (contract or mechanism) exists then there is an agent i whose preference dictates all the preferences. However, this is a contradiction with the former assumption.

For elegant proofs see Schmeidler, Sonnenshein (1978), and Barbara (1983). For more discussion see Kreps (1990) ch. 18.

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